Exercise

Line-clipping (Cohen-Sutherland)

Let a line be defined between (-90,100) and (90,10). Determine which bit-pattern is set for the starting-point and ending-point and clip the line against the visible rectangle which is defined between x in [30,100) and y in [20,80).

Solution:

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bit 4: y >= y_max
bit 3: y < y_min
bit 2: x >= x_max
bit 1: x < x_min
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p_start = 1001
p_end = 0100

1001 & 0100 == 0 so trivially reject or accept is not possible.

$$x = x_0 + t(x_1 - x_0) \Leftrightarrow t = \frac{x - x0}{x_1 - x_0} \qquad \text{(I)}$$

$$x = -90 + 180\,t \Leftrightarrow t = \frac{x + 90}{180}$$

$$y = y_0 + t(y_1 - y_0) \Leftrightarrow t = \frac{y - y0}{y_1 - y_0} \qquad \text{(II)}$$

$$y = 100 - 90\,t \Leftrightarrow t = \frac{y - 100}{-90}$$

Exercise

Line-clipping

Solution:

(1) We consider the starting-point (-90,100) first.

Bit 4 is set => intersect the line with y_max = 80 (use equation II).

$$t = \frac{80 - 100}{-90} = 2/9$$

t in equation I:

$$x = -90 + 180\frac{2}{9} = -90 + 40 = -50$$

$$x = x_0 + t(x_1 - x_0) \Leftrightarrow t = \frac{x - x0}{x_1 - x_0} \qquad \text{(I)}$$

$$x = -90 + 180\,t \Leftrightarrow t = \frac{x + 90}{180}$$

$$y = y_0 + t(y_1 - y_0) \Leftrightarrow t = \frac{y - y0}{y_1 - y_0} \qquad \text{(II)}$$

$$y = 100 - 90\,t \Leftrightarrow t = \frac{y - 100}{-90}$$

The new starting-point becomes (-50,80). In addition we can clear the last bit in 1001 which yields 0001.

Exercise

Line-clipping

Solution:

(2) Bit 1 is set => intersect the line with x_min = 30.

$$t = \frac{30+90}{180} = 120/180 = 2/3$$

t in equation II:

$$y = 100 - 90\frac{2}{3} = 40$$

$$x = x_0 + t(x_1 - x_0) \Leftrightarrow t = \frac{x - x0}{x_1 - x_0} \qquad \text{(I)}$$

$$x = -90 + 180\,t \Leftrightarrow t = \frac{x+90}{180}$$

$$y = y_0 + t(y_1 - y_0) \Leftrightarrow t = \frac{y - y0}{y_1 - y_0} \qquad \text{(II)}$$

$$y = 100 - 90\,t \Leftrightarrow t = \frac{y-100}{-90}$$

The new starting-point becomes (30,40). In addition we can clear the last bit in 0001 which yields 0000. The starting-point is now ready for display.

Exercise

Line-clipping

Solution:

(3) We consider the ending-point (90,10).

Bit 3 is set => intersect the line with y_min = 20 (use equation II).

$$t = \frac{20 - 100}{-90} = 8/9$$

t in equation I:

$$x = -90 + 180\frac{8}{9} = 70$$

$$x = x_0 + t(x_1 - x_0) \Leftrightarrow t = \frac{x - x0}{x_1 - x_0} \qquad \text{(I)}$$

$$x = -90 + 180\,t \Leftrightarrow t = \frac{x + 90}{180}$$

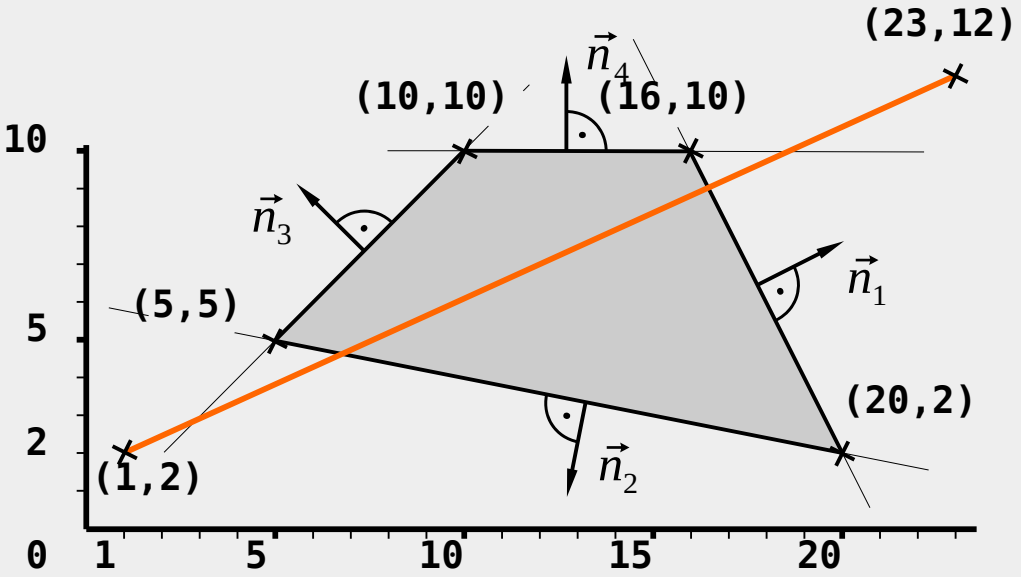
$$y = y_0 + t(y_1 - y_0) \Leftrightarrow t = \frac{y - y0}{y_1 - y_0} \qquad \text{(II)}$$

$$y = 100 - 90\,t \Leftrightarrow t = \frac{y - 100}{-90}$$

The new ending-point becomes (70,20). We also clear the third bit in 0100. Both bit-patterns are 0000, the line can now be drawn safely.

Line clipping

1) A viewing area is given by a convex polygon with vertices **(5,5)**, **(10,10)**, **(16,10)**, **(20,2)**. A line from **(1,2)** to **(23,12)** is to be clipped against the polygon. Perform line clipping according to the Cyrus-Beck approach.



Solution:

We determine normal vector n_1:

n_1 is perpendicular to the line between **(16,10)** and **(20,2)**. The vector in between is

$$\begin{pmatrix}16\\10\end{pmatrix}-\begin{pmatrix}20\\2\end{pmatrix}=\begin{pmatrix}-4\\8\end{pmatrix}$$

We can swap the two components and multiply one by (-1) to get a valid normal vector:

$$\vec{n}_1=\begin{pmatrix}8\\4\end{pmatrix}$$

Line clipping

1) Cyrus-Beck line clipping (cont.):

We now state the boundary line between **(16,10)** and **(20,2)** in normal form:

$$\left[\vec{x}-\begin{pmatrix}20\\2\end{pmatrix}\right]\begin{pmatrix}8\\4\end{pmatrix}=0$$

(20,2) is an arbitrary point on the line. Any point x on the line will yield 0 any other point will produce a non-zero result. Next we state the (orange) line to draw in parameter form:

$$\vec{x}(t)=\begin{pmatrix}1\\2\end{pmatrix}+t\left[\begin{pmatrix}23\\12\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right]=\begin{pmatrix}1\\2\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}$$

The parameter t leads us for t=0 from the first point (1,2) to the end point (23,12) for t=1. We are interested in the t for the intersection with the boundary line. This is done by substituting the parameter form into the normal form:

$$\left[\begin{pmatrix}1\\2\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}-\begin{pmatrix}20\\2\end{pmatrix}\right]\begin{pmatrix}8\\4\end{pmatrix}=0 \quad => \quad \left[\begin{pmatrix}-19\\0\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\right]\begin{pmatrix}8\\4\end{pmatrix}=0$$

Note that the result of the equation will be zero only for one point (namely the intersection between the two lines).

Line clipping

1) Cyrus-Beck line clipping (cont.):

Note that the result of the equation will be zero only for one point (namely the intersection between the two lines).

$$\left[\begin{pmatrix}-19\\0\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\right]\begin{pmatrix}8\\4\end{pmatrix}=\begin{pmatrix}-19\\0\end{pmatrix}\begin{pmatrix}8\\4\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\begin{pmatrix}8\\4\end{pmatrix}=-19x\,8+t\left(22\,x\,8+10\,x\,4\right)=-152+t\,216=0$$

$$t=152/216=19/27$$

We check our result by looking at the sketch: t = ca. 2/3. So travel 2/3 of the line from (1,2) to (23,12). Since we reach the intersection, the result is sound.

Hint: The normal vector (8,4) is quite large. The normal equation for lines works with any length. So (4,2) or (2,1) would also have been fine. Short normals ensure smaller numbers in the calculation.

We apply the same procedure to the remaining three boundary lines.

Line clipping

1) Cyrus-Beck line clipping (cont.):

For normal n_2: The vector between (5,5) and (20,2) is (15,-3) thus e.g.,: $\vec{n}_2 = \begin{pmatrix} -3 \\ -15 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$$\left[\begin{pmatrix}1\\2\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}-\begin{pmatrix}5\\5\end{pmatrix}\right]\begin{pmatrix}-1\\-5\end{pmatrix}=0 \Rightarrow \left[\begin{pmatrix}-4\\-3\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\right]\begin{pmatrix}-1\\-5\end{pmatrix}=0 \Rightarrow 4+15+t(-22-50)=0 \Rightarrow t=19/72$$

For normal n_3: The vector between (5,5) and (10,10) is (5,5) thus e.g.,: $\vec{n}_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$\left[\begin{pmatrix}1\\2\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}-\begin{pmatrix}5\\5\end{pmatrix}\right]\begin{pmatrix}-1\\1\end{pmatrix}=0 \Rightarrow \left[\begin{pmatrix}-4\\-3\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\right]\begin{pmatrix}-1\\1\end{pmatrix}=0 \Rightarrow 4-3+t(-22+10)=0 \Rightarrow t=1/12$$

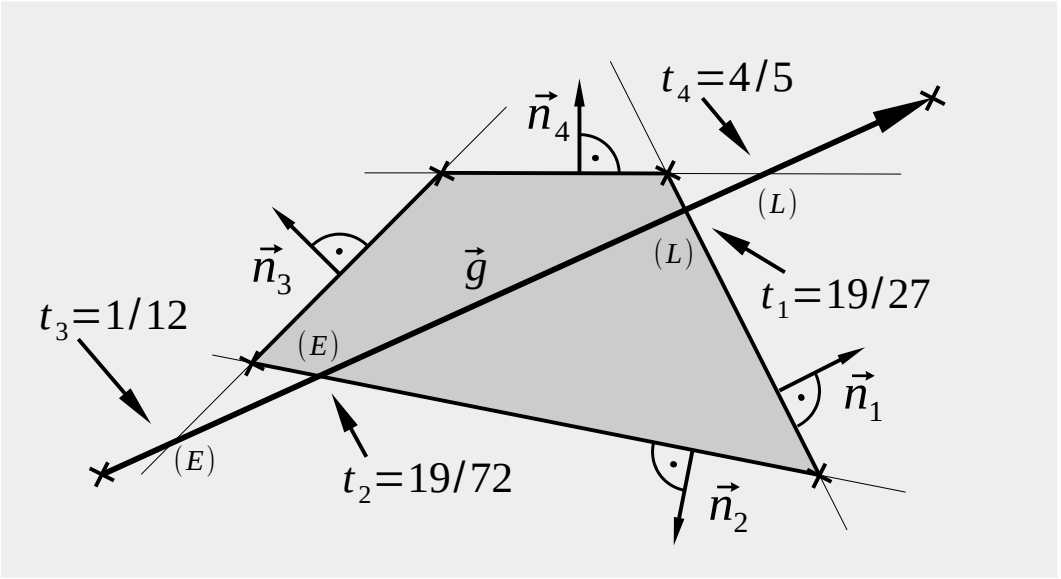
For normal n_4: The vector between (10,10) and (16,10) is (4,0) thus e.g.,: $\vec{n}_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\left[\begin{pmatrix}1\\2\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}-\begin{pmatrix}10\\10\end{pmatrix}\right]\begin{pmatrix}0\\1\end{pmatrix}=0 \Rightarrow \left[\begin{pmatrix}-9\\-8\end{pmatrix}+t\begin{pmatrix}22\\10\end{pmatrix}\right]\begin{pmatrix}0\\1\end{pmatrix}=0 \Rightarrow -8+10t=0 \Rightarrow t=4/5$$

Line clipping

1) Cyrus-Beck line clipping (cont.):

The sketch on the right shows all 4 intersections, but which are the two intersections of the visible line segment?
We need to label every intersection as entering (E) or as leaving (L).
The visible line segment starts with the entering intersection with the largest t and ends with the leaving intersection with the smallest t.



Remember: $\vec{g}\,\vec{n}<0$ means we are entering the viewing area

$\vec{g}\,\vec{n}>0$ means we are leaving.